METACOGNITIVE MODEL IN MATHEMATICAL PROBLEM SOLVING

Leonila B. Barbacena, Ph.D.
Bicol University Tabaco Campus

Norina R. Sy, Ed.D.
Bicol University College of Science, Legazpi City 4500

Introduction

In this era of scientifically advanced world, one may be lost in its ultra modernity. To be abreast of technological booms, one should keep pace with the changes. And the educational system of the twenty-first century is in the midst of the most serious and promising reform effort. Concerted efforts are being forged toward the restructuring the system.

In Mathematics in particular, attempt is being pursued on many fronts namely recognizing that many ways exist to learn and understand Mathematics like using technology to support its learning and connecting Mathematics to the experiences that the students have outside the school (Cusco, 1995). Students are given ample opportunities to explore, investigate and discover for themselves patterns and ideas and even algorithms.

Despite efforts to undergo reform in this field, the stark reality remains that Mathematics performance of students seemed to come short of the standard. The task then presented to the teacher is for him to harness all possible means for maximum learning and development of students. To enable the students to succeed in life after school, there is a need to prepare them to use, understand, control and modify the learning acquired suited to the present situation. The current need for instruction of thinking skills is partly the result of the growing awareness that society's needs and demands are changing. Poor learning is due to learning habits that preclude adequate metacognition (Baird, 1998). This expectation means that there is a need to develop genuinely mathematical ways of thinking. The thinking process, which is the key in learning, can be controlled and directed.

Having known that thinking can be controlled and directed, students will be conscious of their own powers of thinking and be a purposeful learner. Schools commonly espouse the aim to educate students to be aware, responsible and capable of independent learning (Baird, 1998). And if they are developed metacognitively, they will earn confidence that they can learn, will know how to assess their stand in learning situation and will be able to view themselves as continual learners and thinkers.

Statement of the Problem

The main focus of this study is to build a theoretical model of metacognition in Mathematics within the context of problem solving among the junior Mathematics major students of Bachelor of Secondary Education of Bicol University College of Education. The study sought to provide answer to the following questions:

1. What thinking skills do individual learners utilize as they go around thinking about problem solving?

2. What pattern of thinking process can be drawn relative to the responses?

3. What pattern of metacognition is exhibited by the students in the context of metacognitive functions?

4. What metacognitive model of learning can be abstracted based on the data gathered?

Related Literature

Metacognition is often described as multi-dimensional and has been used as a general term about a range of disparate higher level of cognitive skills (Thorpe and Satterly, 1990). A common definition is "thinking about thinking" (Babbs and Moe, 1983). Metacognition is one's ability to know what he knows and what he does not know (Seastone, 1994). It is also the ability to use one's own prior knowledge to plan a strategy for producing information, to take necessary steps in problem solving and to reflect on the quality of one's thinking about a particular concern.

Varying definitions of metacognition exist in the literature but generally they include a number of components that which are interrelated (Schraw and Dennison, 1994). Within the range of definitions, it is generally agreed that metacognition involves two components; knowledge about cognition and regulation of cognition (Brown, 1987, Ferrara and Campione, 1983, Garofalo and Lester, 1984, Schoenfeld, 1990 and Schraw and Dennison, 1994) but the nature of the relationship is not always clearly defined either.

There are three ways to talk about metacognition in the language of mathematics: beliefs and intuitions, knowledge and self-awareness (Seastone, 1994). Schoenfeld (1992) suggested that one could think of this as management approach. Aspects of management includes (a) making sure that one understands the problem before one hastily attempts a solution; (b) planning; (c) monitoring or keeping track of how well things are going during a solution and (d) allocating resources, or deciding what to do, and for how long one takes to work on the problem.

What is basic to the concept of metacognition is the notion of thinking about one's own thoughts. Those thoughts can be about what one knows, i.e. metacognitive knowledge; what one is currently doing i.e. metacognitive skill; or what one's current cognitive or affective state is i.e. metacognitive experience.

Analytic Framework

Metacognitive thoughts do not spring from a person's immediate external reality; rather, their source is tied to the person's own internal mental representations of that reality, which can include what one knows about that internal representations, how it works, and how one feels about it.

The most important factor in cognitive growth is the process of equilibrium or self-regulation. Synthesis based on the literature cited brings a definition of metacognition process as used in this study. Metacognition refers to the awareness individuals have of their own thinking and their evaluation and regulation of their own thinking. Three metacognitive functions are used to describe student behavior such as metacognitive awareness (MA), metacognitive evaluation (ME), and metacognitive regulation (MR). Metacognitive Awareness relates to an individual's awareness of where they are in the learning process, their own knowledge about content knowledge, personal learning strategies, and what has been done and needs to be done. Metacognitive Evaluation refers to the judgment made regarding one's thinking capacities and limitations as they are employed in a particular situation or as a self attributes. Metacognitive Regulation occurs when individuals modify their thinking.

Student thinking skills were considered and matched according to Marzano's (1998) definition of thinking skills. Marzano's definition of thinking skills include defining problem, setting goals, observing, formatting questions, encoding, recalling, comparing, classifying, ordering, representing, identifying, inferring, predicting, elaborating, summarizing, restructuring, establishing criteria, and verifying. Based on the use of these cognitive skills and metacognitive functions shown, a metacognitive model of learning Mathematics in the context of problem solving was abstracted. After identifying the thinking skills used, a paradigm of thinking process was deducted. Vis-à-vis, each thinking skill was analyzed further in terms of metacognitive functions employed as MA, ME, MR.

Method

This study is a qualitative type of research intertwined with both the methods of exploratory and phenomenological design. The exploratory case method is implemented in a narrative review by treating each document as a data point that must be 'scrutinized, summarized, and utilized' in a relatively unencumbered but systematically exhausted research for emergent themes and pattern. Phenomenological research emphasizes the individual's subjective experience (Tesch, 1990). According to Mertens (1998), phenomenological design seeks the individual perception and meaning of a phenomenon or experience. Inasmuch as the purpose of this study is to describe the step-by-step procedure of individual's thinking as they go about the process of problem solving, descriptive method is also considered.

The Respondents

The respondents of this study were the eighteen third year Mathematics major students of Bachelor of Secondary Education of Bicol University College of Education. The class was composed of four males and fourteen females divided into ability grouping namely Group A and Group B, the former group is deemed as the higher ability group and the latter the lower ability group based on their numerical ability result and average grade in the Mathematics subjects taken.

The Instrument

The instrument consisted of four problem solving questions, of which one is lifted from Sourcebook on Practical Work for Teacher Trainers, two are taken from the compilation of Resource Materials in Rescue Initiatives for Science Education (RISE), and one is constructed by the researcher. The four problems had reference to Algebra, Geometry, Statistics and Arithmetic.

Data Collection Procedure

The respondents were made to answer the problems in two modes. The first style of gathering data was administering the first two questions to all students at the same time. Then, they were asked to write down their own reflections as to what went on their minds as they were answering the questions. In the second mode of gathering data, each student was first asked to answer the last two problems and was interviewed to reflect on thinking skills he had while answering the question. During the one-on-one interview, however, the respondents were just advised by the researcher to describe in sequence what went on in their minds as they were answering the questions. Thereby also the researcher noted the behavior of the subjects. In this aspect of the data gathering, the researcher herself functioned as a key instrument (Bogdan and Biklar, 1992). The interviews conducted were all recorded. So as to elicit or to approximate the exact working of the student's mind, the account as to how they thought about thinking the problem was not limited to the English language. Student's reflections were mixed some preferred Filipino, others opted for English and there were few who resorted to the Bicol dialect.

Data Analysis

Data analysis in qualitative studies is an ongoing process (Mertens, 1998). As such, analysis of the data cannot be described sequentially, moreover, the following steps are considered (Miles and Huberman, 1994):

a. Give codes to the notes drawn from observation, interview or answer sheet.
b. Note personal reflections and other comments on margins.
c. Sort and sift through the materials to identify similar phrases, relationships between variables, patterns, describe differences between subgroup and common sequences.
d. Identify patterns and processes, commonalities and differences and take them out to the field in the next round of data collection.
e. Begin elaborating a small set of generalization that cover the consistencies discovered in the database.
f. Examine known generalization in the light of a formalized body of knowledge in the form of constructs and theories.

While the enumerated steps suggested and guided the researcher, nonetheless she was forced to resort to exploratory data analysis given the nature of data at hand. The written solutions was analyzed as to its quality. Transcript of the recorded interviews and written reflections were the basis for determining the thinking skills of the students. The thinking skills were not explicitly pronounced by the respondents, but were inferred by the researcher using Marzano's definition.

The oral and written reflections were further analyzed for their metacognitive actions by the aide of metacognitive cue card. As the students were asked to reflect on their thinking as went around thinking the problems, their metacognitive actions were noted and categorized into three metacognitive functions as metacognitve awareness (MA), metacognitive evaluation (ME) and metacogntive regulation (MR).

Discussion

For the problem number 1: *Verify and explain why the square of an odd number always leaves as remainder of one when divided by eight*, the thinking skills utilized by the learners were recalling (Rec), representing (Rep), identifying relation (IR), and elaborating (Ela). For problem number 2: *A candy recipe needs ½ cup each of milk, butter and nuts for every 3 cups of sugar. What fractional part of the recipe is butter?*, the thinking skills used were defining the problem (Def), representing, establishing criteria (Esc) and setting goals (Seg). For the third problem solving, *Mr. Castro is scheduling a basketball tournament in their barangay. There are five teams that registered. Each team will play with all the other teams once. How many games will be scheduled?*, the thinking skills used by the respondents were defining the problem (Def), representing (Rep), comparing (Co), identifying relations (IR) and verifying (Vr). For the fourth problem: Consider forming a flow chart with an input of x and an output of 24 with any two operations add 4, subtract 6, multiply by 8, and divide by 10,, the thinking skills of defining the problem (Def), setting goals (Seg), representing (Rep), verifying (Vr), and identifying relations (IR) were used. Representing and identifying relations were the commonly used thinking skills in solving problems. As revealed by the study, problem solving generally starts with defining the problem. By the use of this skill, the students are helped and guided to identify the difficult patterns. Respondents who belonged to higher ability group utilized more thinking skills than those students in lower ability group had.

There were four patterns of thinking processes exhibited by the students relative to problem solving 1, namely RecRep, RecRepIR, RecRepEla, and RepEla. For the problem solving 2, the patterns were DefRep, DefEscRep, DefRecRep and DefSegRecRep. In problem solving 3 the learners had DefRep, DefRepCo, DefIR, and DefRepIRVr. Based on problem solving 4, learners had five patterns of thinking process namely DefSegRep, DefSegRepVr, DefSegRepIRVr, DefSegRepIRRep, and DefSegRepIRVrRep. It can be observed that representing skill serves as terminal skill in most of the pattern. This is followed by verifying skill. It points to the fact that students try to prove the correctness of their answers as they work on problems.

There were six patterns of metacognitive actions the students have shown. These were MA-ME, MA-ME-MR, MA-MR, MA-MR-ME, MR-MA-ME, ME-MR. See Figure 1. As can be seen, all these metacognitive actions were employed by the students in solving problems. Among the patterns, the generally practiced pattern is MA-ME which was exhibited 46 times, followed by MA-ME-MR which occurred 14 times.

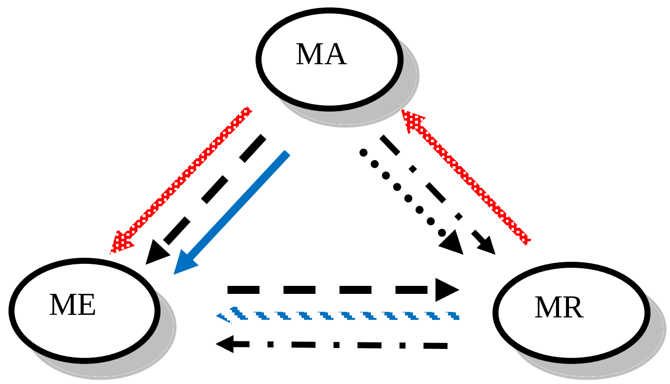

Figure 1. Metacognitive Functions in Problem Solving

The students are sort of metacognitive evaluative (ME) rather than metacognitive regulative (MR). The inclination to be conscious of the outcome of any undertaking can be inferred to be influenced by the symptom of being product oriented in the school environment. All across discipline the emphasis is output. Seldom does the emphasis shifted from product to the process. While the paradigm in Mathematics seems to be weighed equally on product and process, reality points to the opposite. There is an imbalance between those two aspects, the former being weighed more heavily than the latter. This accounts for more ME behavior than MR behavior. As revealed by the study, barely half of the respondents completely utilized MA, ME, MR. Of these number, five students in group A, meaning all those who belonged to the higher ability group demonstrated wholly the different metacognitive functions. It is worth mentioning that all those students who capitalized on the three functions resulted to correct responses, except one.

Within each metacognitive function are exit points or entry points that lead to such activity, these are the thinking skills or cognition. As noticed by the researcher, before metaognitive awareness operated a thinking skill preceed to aid MA. In the same manner cognitive activity happened before or after metacogntive evaluation or metacognitive regulation took place. Metacognitve experiences usually precede or follow a cognitive activity (Roberts & Erdos, 1993). This supports the claim that metacognition and thinking skills are inseparable. One could never account what went on his mind without reference to any thinking skills. From the foregoing, the researcher conceptualized a model (See Figure 2) of metacognitive functions vis-a-vis the thinking skills (TS). From the said model, problem solving becomes an easy task if learners are able to know the content knowledge relative to the problem, that is being aware of the learning situation. But prior to solve a problem, they have to use their repertoire of thinking skills. Dealing with a problem calls for awareness on the part of the learner. Such awareness involves what the problem is all about. The learners should involve their own knowledge of the task and the interaction of their set of thinking skills. Reflection, or thinking about the thinking that transpired needs to be constantly employed. That students have to regulate or monitor their own thinking through the aid of thinking skills may look repetitive, but it has to be that way. The end view of the model is to come up with a thinking learner who is able to command his thoughts and actions thus capable of independent learning.

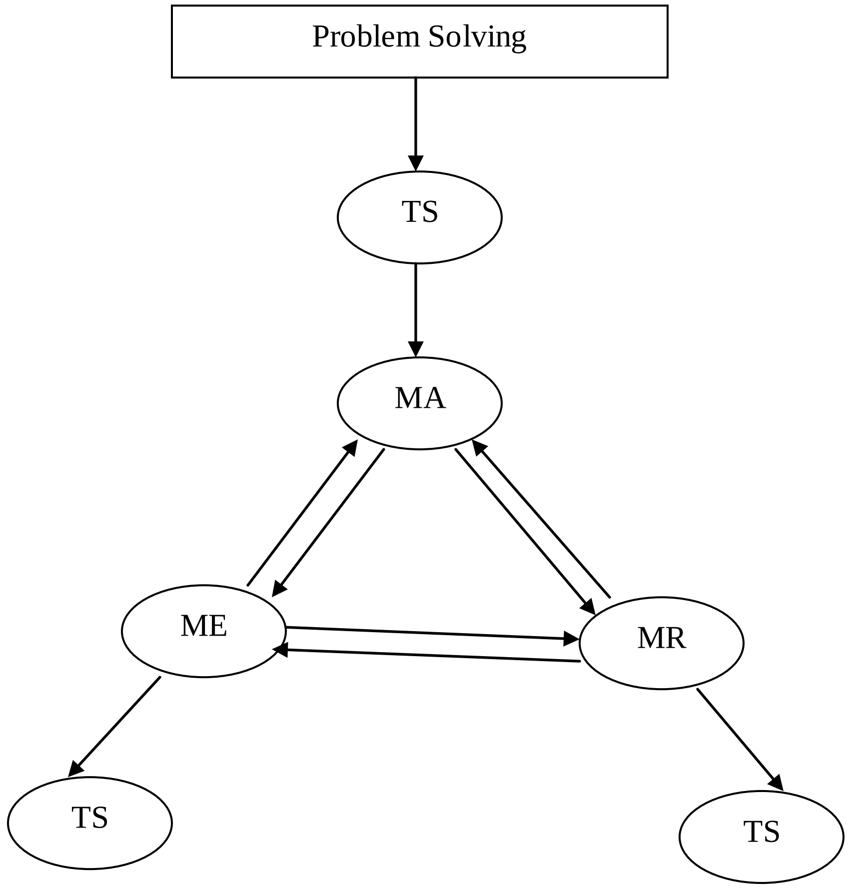

Figure 2. Metacognitive Model in Problem Solving

Conclusion

1. The learners were found to use minimal repertoire of thinking skills and found not to manifest other thinking skills.
2. The patterns of thinking processes is at least made up of two thinking skills and at most comprised of five thinking skills. The said patterns seemed to conclude that thinking process is influenced by the ability grouping. The study supports that there is no such thing as best thinking process for any pattern resulted to both correct and wrong response.
3. There was a general tendency among subjects to be metacognitive evaluative in their behavior rather than metacognitive regulative. Students who utilized metacognitive regulation in their solution more often resulted to correct response than those who did not employ this function. The study gave the insight that subjects who capitalized on metacognitive functions resulted to better quality of answer. Thus it was successful in bringing up the concern that metacognition can be capitalized for better learning.

Recommendations

In view of the relative usefulness of metacognition in learning of Mathematics glimpsed from the study, the following recommendations are advanced.

1. As students were found to exhibit limited thinking skills, teachers need to teach thinking skills deliberately.
2. Teachers should use innovative strategies like reflective journal writing, cooperative learning to enhance and develop students' own thinking skills and how to direct their own thinking.
3. A replication of the study on metacognition to a cross-sectional group of students of large sample to establish a pattern between and among thinking skill, ability grouping and quality of answer.

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